HW9 Solutions

Chapter 10

Prob 12.

$T = 960K \Rightarrow kT \simeq 0.0827\,eV$

So, $\dfrac{N_2}{N_1} = \dfrac{d_2 e^{-E_2/kT}}{d_1 e^{-E_1/kT}} = 3e^{-(E_2 - E_1)/kT} \simeq 0.146$

With, $N = N_1 + N_2$, we can get $N_1 \simeq 0.87N$, $N_2 \simeq 0.13N$.

Prob 14.

v_p occurs at $N(v)$ maximum. So,

$$\frac{dN}{dv} = 0 \Rightarrow 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}[2v + v^2(-2mv/2kT)]e^{-mv^2/2kT} = 0$$

$$\Leftrightarrow 2v - \frac{mv^3}{kT} = 0$$

$$\Leftrightarrow v_p = \sqrt{\frac{2kT}{m}}$$

Prob 15.

$T = 293K \Rightarrow kT \simeq 0.02525\,eV$

(a) $E_m = \dfrac{3}{2}kT \simeq 0.0379\,eV$

(b) $$dN = N(E_m)\,dE = \frac{2N_A}{\sqrt{\pi}\,(kT)^{3/2}}\sqrt{E_m}\,e^{-E_m/kT}dE$$
$$\simeq 2.78 \times 10^{21}$$

Prob 25.

$E_F = 3.00\,eV$

(a) $T = 295K \Rightarrow kT \simeq 0.0254\,eV$

So, $$\frac{N(E)\,dE}{V} = \frac{8\sqrt{2}\,\pi m^{3/2}}{h^3}\sqrt{E}\,\frac{1}{e^{(E-E_F)/kT}+1}dE$$
$$\simeq 1.03 \times 10^{-7}\,m^{-3}$$

(b) $T = 2500K \Rightarrow kT \simeq 0.215\,eV$

So, $$\frac{N(E)\,dE}{V} = \frac{8\sqrt{2}\,\pi m^{3/2}}{h^3}\sqrt{E}\,\frac{1}{e^{(E-E_F)/kT}+1}dE$$
$$\simeq 1.48 \times 10^{23}\,m^{-3}$$

Prob 27.

(a) Mass of star $M = N \times m_n$ (m_n : mass of neutron)

$$E_{grav} = -\frac{3}{5}\frac{GM^2}{R} = -\frac{3}{5}\frac{GN^2 m_n^2}{R}$$

$$E_{neut} = NE_m = \frac{3}{5}NE_F = \frac{3}{5}N\frac{h^2}{2m_n}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

$$E = E_{grav} + E_{neut}$$

Minimum radius occurs at $dE/dR = 0$, so, we can get the radius solving this :

$$R = \frac{h^2}{GN^{1/3}m_n^3}\left(\frac{9}{32\pi^2}\right)^{2/3}$$

(b) $M_{sun} \simeq 2.00 \times 10^{30}\,kg$, so $M = 3M_{sun} \simeq 6.00 \times 10^{30}\,kg$

& $N = \frac{M}{m_n} \simeq 3.6 \times 10^{57}$

So, $R \simeq 8.6\,km$

(c) Density $\rho = \frac{M}{V} \simeq 2.2 \times 10^{18}\,kg/m^3$

Prob 28.

(a) $M_{sun} \simeq 2.00 \times 10^{30}\,kg$, so $M = 2M_{sun} \simeq 4.00 \times 10^{30}\,kg$

& $N = \frac{M}{m_n} \simeq 2.4 \times 10^{57}$

Fermi energy

$$E_F = \frac{h^2}{2m_n}\left(\frac{3N}{8\pi V}\right)^{2/3} \simeq 140\,MeV$$

This value is small compared to the rest energy of the neutron($940\,MeV$), so we can use classical kinematics.

(b) The de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \simeq 2.4\,fm$$

Also, from $R = \frac{h^2}{GN^{1/3}m_n^3}\left(\frac{9}{32\pi^2}\right)^{2/3} \simeq 9.8\,km$, we can get neutron number density

$$\frac{N}{V} \simeq 6.1 \times 10^{44}\,m^{-3}$$

So, average neutron spacing is $\left(\frac{N}{V}\right)^{-1/3} \simeq 1.18\,fm$

de Broglie wavelength is about two times larger than average neutron spacing.