

HW7 Solutions

Prob 5.

With $l = 2$, $\cos\theta = \frac{m_l}{\sqrt{l(l+1)}} = \frac{m_l}{\sqrt{6}}$.

m_l	$\theta(^\circ)$
2	35
1	66
0	90
-1	114
-2	145

Prob 7.

(a) $n = 6$ with $l_{\max} = n - 1$

$\Rightarrow l = 0, 1, 2, 3, 4, 5$

(b) $m_l = -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6$

(c) 5

(d) 4

Prob 9.

From the table, $\psi_{200} = \frac{1}{\sqrt{4\pi}(2a_0)^{3/2}}(2 - \frac{r}{a_0})e^{-r/2a_0}$, substituting into eq (7.10) :

$$-\frac{\hbar^2}{2m} \left[\frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial \psi}{\partial \theta}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2} \right] + U(r)\psi = E\psi$$

with $U(r) = -\frac{e^2}{4\pi\epsilon_0 r}$,

we can get $\frac{e^2}{4\pi\epsilon_0}(-\frac{1}{8a_0})\psi_{200} = E\psi_{200}$.

So, $E_{200} = \frac{1}{4}(-\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2})$: first excited state energy of hydrogen

From the table, $\psi_{210} = \frac{1}{\sqrt{4\pi}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \cos\theta$, substituting into eq (7.10),

we can get $\frac{e^2}{4\pi\epsilon_0}(-\frac{1}{8a_0})\psi_{210} = E\psi_{210}$

So, $E_{210} = \frac{1}{4}(-\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2})$: first excited state energy of hydrogen

Prob 12.

For 1s level, $P(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0}$.

$$\frac{dP}{dr} = 0 \Rightarrow \frac{8r}{a_0^3} e^{-2r/a_0} \left(1 - \frac{r}{a_0}\right) = 0 \text{ which has three solutions}$$

$$\therefore r = 0, a_0, \infty$$

substituting these solutions into $P(r)$, we can find that $P(r)$ has maximum at $r = a_0$

Prob 18.

(a) Degeneracy = $2n^2 = 50$

(b) For each l , degeneracy = $2(2l+1)$

l	Degeneracy
0	2
1	6
2	10
3	14
4	18
total	50

Prob 25.

(a) $3d$ to $2p$ transition, energy difference is

$$E = (-13.6 \text{ eV}) \left(\frac{1}{3^2} - \frac{1}{2^2} \right) = 1.889 \text{ eV}$$

$$\lambda = \frac{hc}{E} = 656.11 \text{ nm}$$

$$\Delta\lambda = \frac{\lambda^2}{hc} \Delta E = 0.07 \text{ nm}$$

So, wavelength of the three normal Zeeman components,

656.04 nm , 656.11 nm , 656.18 nm

(b) Since E only depends on n , the answer is same as (a)

Prob 27.

$3d$ fine structure splitting,

$$\Delta E = mc^2 \alpha^4 \frac{1}{n^5} = mc^2 \alpha^4 \frac{1}{3^5} = 6.0 \times 10^{-6} \text{ eV}$$

Assuming that the parallel and anti-parallel states are affected equally, then $\pm 3.0 \times 10^{-6} \text{ eV}$ is applied.

So, $3d$ fine structure energy should be $-1.51 \pm 3.0 \times 10^{-6} \text{ eV}$.

Similarly, for 2p state, its fine energy should be $-3.40 \pm 2.25 \times 10^{-5} eV$.

So, possible energy differences are $1.89 \pm 2.55 \times 10^{-5} eV$ and $1.89 \pm 1.95 \times 10^{-5} eV$.

From this, $\Delta\lambda = \frac{\lambda^2}{hc} \Delta E = 0.00885 nm$ and $0.00677 nm$.

So, 4 component wavelengths are there :

$656.112 \pm 0.0089 nm$ and $656.112 \pm 0.0068 nm$