HW10 Solutions

Prove $E_F = \frac{\hbar^2 \pi^2}{2m} (\frac{3n_e}{\pi})^{2/3}$.

Number of electrons energy between E & E+dE are

$$N(E)\,dE = \frac{V}{2\pi^2} (\frac{2m}{\hbar^2})^{3/2} \frac{\sqrt{E}\,dE}{e^{\beta(E-E_F)}+1}.$$

So, the total number of particles N is

$$N = \int N(E)\,dE = \frac{V}{2\pi^2} (\frac{2m}{\hbar^2})^{3/2} \int_0^\infty \frac{\sqrt{E}\,dE}{e^{\beta(E-E_F)}+1}.$$

At $T=0$, it becomes $N = \frac{V}{2\pi^2} (\frac{2m}{\hbar^2})^{3/2} \int_0^{E_F} \sqrt{E}\,dE$.

Calculating this, we can get $N = \frac{V}{2\pi^2} (\frac{2m}{\hbar^2})^{3/2} \frac{2}{3} E_F^{3/2}$.

With $\frac{N}{V} = n_e$, we get $E_F = \frac{\hbar^2 \pi^2}{2m} (\frac{3n_e}{\pi})^{2/3}$.

Chapter 11

Prob 14.

$T_D = 225K$

(a) $T = 4K$, heat capacity $C = 0.0134\ J/mol \cdot K$.

Lattice contribution to heat capacity

$$C_L = \frac{12\pi^4 R}{5} (\frac{T}{T_D})^3 \simeq 0.0109\ J/mol \cdot K$$

So, electronic contribution is $C_e = 0.0025\ J/mol \cdot K$.

(b) $T = 2K$

Lattice contribution to heat capacity $C_L \propto T^3$,

$C_L \simeq 0.00136\ J/mol \cdot K$

Also, electronic contribution $C_e \propto T$,

electronic contribution is $C_e = 0.00125\ J/mol \cdot K$.

So, total heat capacity is $C = 0.00261\ J/mol \cdot K$.

Prob 15.

(a) $T = 293K \Rightarrow kT \simeq 0.02525\,eV$

$$f_{FD}(E) = \frac{1}{e^{(E-E_F)/kT}+1} = 0.1$$
$$\Rightarrow E = E_F + kT\ln(9)$$
$$\simeq 7.09\,eV$$

(b) $f_{FD}(E) = \frac{1}{e^{(E-E_F)/kT}+1} = 0.9$
$$\Rightarrow E = E_F - kT\ln(9)$$
$$\simeq 6.98\,eV$$

So, energy range of $0.11\,eV$.

Prob 17.

$T = 293K \Rightarrow kT \simeq 0.02525\,eV$

Eq (10.47) : $dN = V\frac{8\pi\sqrt{2}\,m^{3/2}}{h^3}E^{1/2}\frac{1}{e^{(E-E_F)/kT}+1}dE$

implies $\frac{dN}{V} = \frac{8\pi\sqrt{2}\,m^{3/2}}{h^3}E^{1/2}\frac{1}{e^{(E-E_F)/kT}+1}dE \simeq 2.3\times10^{24}m^{-3}$

Prob 18.

Eq (11.19) : $\sigma = \frac{ne^2\tau}{m}$

implies $\tau = \frac{\sigma m}{ne^2} \simeq 2.50\times10^{-14}s$.

So, mean free path $l = \tau v_F \simeq 39.2\,nm$,

which is 150 times larger than lattice spacing of copper($0.256\,nm$).

Prob 20.

The current density $j = \frac{i}{A} \simeq 1.27\times10^4 A/m^2$.

So, $v_d = \frac{j}{ne} \simeq 9.37\times10^{-7}m/s$.

With $dN = N(E)\,dE \simeq N(E_F)\,dE = N(E_F)\,mv_Fv_d$ and

$N(E_F) = \frac{3}{4}\frac{N}{E_F}$(from eq (10.47) & eq (10.49)),

we have $\frac{dN}{N} = \frac{3}{4}\frac{mv_Fv_d}{E_F} \simeq 8.9\times10^{-13}$.

Prob 22.

$T = 293K \Rightarrow kT \simeq 0.02525\,eV$

$E - E_F = \frac{E_g}{2}$ (by problem assumption)

So,

$$f_{FD}(E) = \frac{1}{e^{(E-E_F)/kT}+1} = \frac{1}{e^{E_g/2kT}+1} \simeq 5.0 \times 10^{-48} \text{ for C}$$

$$f_{FD}(E) = \frac{1}{e^{(E-E_F)/kT}+1} = \frac{1}{e^{E_g/2kT}+1} \simeq 3.5 \times 10^{-10} \text{ for Si.}$$

The ratio of electron concentration is 1.4×10^{-38}.