PH241 Final Solutions

Prob 1 (10pt).

(a) (5pt)

1) $3d$ to $2p$ transition, energy difference is

$$E = (-13.6\,eV)\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.889\,eV$$

$$\lambda = \frac{hc}{E} = 656.11\,nm$$ …… +2pts

$$\Delta\lambda = \frac{\lambda^2}{hc}\Delta E = 0.08\,nm$$ …… +1pt

So, wavelength of the three normal Zeeman components,

$656.03\,nm$, $656.11\,nm$, $656.19\,nm$ …… +1pt

2) Since E only depends on n, 3s to 2p transition is same as 1) …… +1pt

(b) (5pt) There are 6 possible sets for each 2p electrons. …… +2pts So, $6 \times 6 = 36$ possibilities. …… +1pt Also, there are 6 combinations in which the two sets are identical. So, by Pauli principle, $36 - 6 = 30$ possibilities. …… +2pts

Prob 2 (10pt).

(a) (5pt)

$$k = \frac{2(E - E_{\min})}{(R - R_{eq})^2} \simeq 310\,eV/nm^2$$

Also, from $\mu = \dfrac{m(Na)m(Cl)}{m(Na)+m(Cl)} \simeq 13.96u$, …… +2pts

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} \simeq 7.4 \times 10^{12}\,Hz$$

So, $\lambda = \dfrac{c}{f} \simeq 40\,\mu m$ (infrared region) …… +1pt and $E = hf \simeq 0.031\,eV$. …… +1pt

Since it's valid about 0.25eV, maximum vibrational quantum number is 8. …… +1pt

(b) From $\mu = \dfrac{m(Na)m(Cl)}{m(Na)+m(Cl)} \simeq 13.96u$ and $R_{eq} = 0.236nm$, we can get

$$B = \frac{\hbar^2}{2mR_{eq}^2} \simeq 2.68 \times 10^{-5}\,eV.$$ …… +2pts

So,

$L=1$ to $L=0$ (lowest) : $\Delta E = 2B = 5.36 \times 10^{-5}\,eV$ & $\lambda = \dfrac{hc}{\Delta E} \simeq 23.1\,mm$ …… +2pts

$f = \frac{c}{\lambda} \simeq 1.3 \times 10^{10}\, Hz$ (Microwave region) ⋯⋯ +1pt

Prob 3 (10pt).

(a) (5pt) $T = 1000K \Rightarrow kT \simeq 0.0862\, eV$ ⋯⋯ +1pt

So, $\frac{N_2}{N_1} = \frac{d_2 e^{-E_2/kT}}{d_1 e^{-E_1/kT}} = 3e^{-(E_2 - E_1)/kT} \simeq 0.295$ ⋯⋯ +2pts

With, $N = N_1 + N_2$, we can get $N_1 \simeq 0.77N$, $N_2 \simeq 0.23N$. ⋯⋯ +2pts

(b) (5pt) $T = 300K \Rightarrow kT \simeq 0.02585\, eV$ ⋯⋯ +1pt

$E - E_F = \frac{E_g}{2}$ (by problem assumption) ⋯⋯ +1pt

So,

$f_{FD}(E) = \frac{1}{e^{(E-E_F)/kT}+1} = \frac{1}{e^{E_g/2kT}+1} \simeq 6.4 \times 10^{-47}$ for C

$f_{FD}(E) = \frac{1}{e^{(E-E_F)/kT}+1} = \frac{1}{e^{E_g/2kT}+1} \simeq 5.8 \times 10^{-10}$ for Si. ⋯⋯ +2pts

The ratio of electron concentration is 1.1×10^{-37}. ⋯⋯ +1pt

Prob 4 (10pt).

(a) (5pt) Maxwell distribution : distribution of velocity (i.e., $f(v) = C_1 e^{-\frac{\frac{1}{2}mv^2}{kT}}$ ⋯⋯ +2pts)

Boltzmann distribution : distribution of energy (i.e., $f(E) = C_2 e^{-\frac{E}{kT}}$ ⋯⋯ +2pts)

+1pt for mention the velocity & energy

(b) (5pt) For number density distribution of energy $n(E)$,

for collision $1+2 \rightarrow 3+4$, rate of collision $R(1+2 \rightarrow 3+4) = C_1 n(E_1) n(E_2)$ is satisfied.

And for collision $3+4 \rightarrow 1+2$, $R(3+4 \rightarrow 1+2) = C_2 n(E_3) n(E_4)$ is satisfied. ⋯⋯ +1pt

From micro-reversibility, we can get $C_1 = C_2$, and so $n(E_1)n(E_2) = n(E_3)n(E_4)$. ⋯⋯ +1pt

Take the logarithm, we can get $\ln n(E_1) + \ln n(E_2) = \ln n(E_3) + \ln n(E_4)$.

With energy conservation $E_1 + E_2 = E_3 + E_4$ ⋯⋯ +1pt, we can guess $n(E)$ should be the form of $\ln n(E) = \alpha - \beta E$.

Equivalent to $n(E) = Ce^{-\beta E}$ …… +1pt

Since when $E = \frac{1}{2}mv^2$ (free particle energy), $n(E)$ should be same as Maxwell distribution, so we can find that $\beta = \frac{1}{kT}$. …… +1pt

So, $n(E) = Ce^{-\frac{E}{kT}}$.

Prob 5 (10pt).

(a) (4pt) Mass of star $M = N \times m_n$ (m_n : mass of neutron)

$$E_{grav} = -\frac{3}{5}\frac{GM^2}{R} = -\frac{3}{5}\frac{GN^2m_n^2}{R}$$ …… +1pt

$$E_{neut} = NE_m = \frac{3}{5}NE_F = \frac{3}{5}N\frac{h^2}{2m_n}\left(\frac{3N}{8\pi V}\right)^{2/3}$$ …… +1pt

$$E = E_{grav} + E_{neut}$$

Minimum radius occurs at $dE/dR = 0$, so, we can get the radius solving this :

$$R = \frac{h^2}{GN^{1/3}m_n^3}\left(\frac{9}{32\pi^2}\right)^{2/3}$$ …… +2pts

(b) (4pt) $M_{sun} \simeq 2.00 \times 10^{30}\,kg$, so $M = 4M_{sun} \simeq 8.00 \times 10^{30}\,kg$

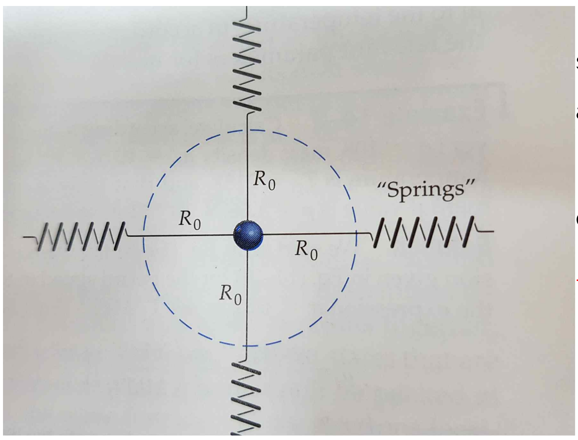
& $N = \frac{M}{m_n} \simeq 4.8 \times 10^{57}$ …… +2pts

So, $R \simeq 7.8\,km$ …… +2pts

(c) (2pt) Density $\rho = \frac{M}{V} \simeq 3.98 \times 10^{18}\,kg/m^3$ …… +2pts

Prob 6 (10pt).

(a) (5pt)



Let the amplitude R_0, then the scattering cross section $\sigma = \pi R_0^2$ can be assumed. …… +1pt

Also, for this model, we can get kinetic energy $E_{kinetic} = \frac{1}{2}mv^2 = \frac{1}{2}mR_0^2w^2$. …… +1pt

Using the equipartition theorem, we get $\frac{1}{2}mR_0^2w^2 = \frac{1}{2}kT$. …… +1pt

So, $R_0^2 = \frac{kT}{mw^2}$.

And finally we get $\sigma = \pi \frac{kT}{mw^2}$. …… +2pts

(b) (5pt) Since $\sigma \propto T$ …… +1pt and other terms in ρ are temperature independent …… +2pts. So, we can easily check that $\rho \propto \sigma \propto T$ …… +2pts.

Prob 7 (10pt).

Heat capacity $C = C_{el} + C_{ph}$

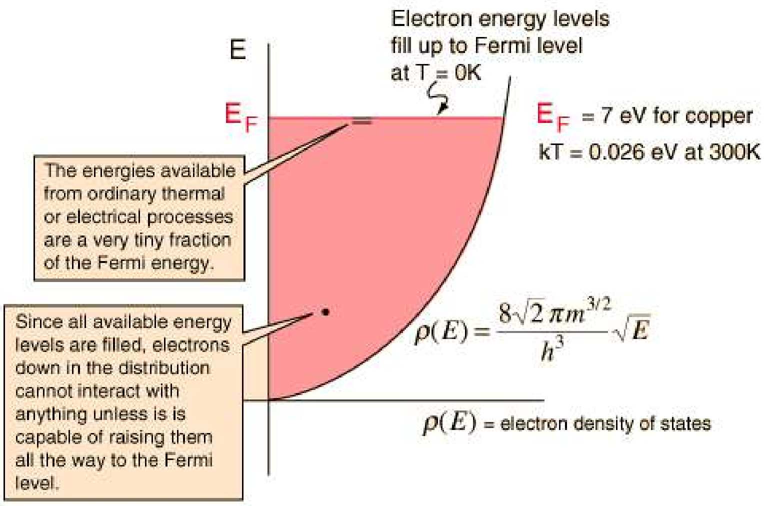
where C_{el} : contribution from electron and C_{ph} : contribution from phonon.

1) C_{el}

Using the Fermi-Dirac distribution ($N(E) = \frac{2}{e^{(E-E_F)/kT}+1}$), we can find the

$$< E > = \frac{1}{N}\int EN(E)\,dE \propto T^2.$$

Also, from the definition of heat capacity $C \equiv \frac{\partial U}{\partial T}$, we can get $C_{el} \propto \frac{T}{T_F}$.



For this C_{el}, since $k_B T$(order of meV) is very small compared to E_F(order of few eV) of metals, only small number of electrons near E_F can contribute to C_{el}. So, it is very small.

2) C_{ph}

Since phonon is boson, quantum mechanically at low temperature, we have to use

Bose-Einstein statistics($f_{BE}(E) = \frac{1}{A_{BE}e^{E/kT} - 1}$). …… +3pts

Also, phonon has vibrational energy of $\hbar w_{ph}$(order of 10meV) and at low temperature about 15K or below, $k_B T$ ~ $1meV$ or lower, so it should be $f_{BE}(E) \sim e^{-\frac{\hbar w_{ph}}{kT}}$. …… +5pts

[Using the Debye model(considering solid as a ensemble of independent of quantum harmonic oscillators), we can get $C_{ph} \propto (\frac{T}{T_D})^3$.]

So, at low temperature($\hbar w_{ph} \gg k_B T$), thermal energy is too small to excite the phonon and equipartition theorem can not be applied, phonon contribution becomes also very small.

So, for $C = C_{el} + C_{ph}$, C becomes very small when $T \rightarrow 0K$. …… +2pts.